Parametrisierungen in der Ebene

wichtig für uns sind i. W.:

(i) Strecken zwischen 2 Punkten $A = \begin{pmatrix} a_x \\ a_y \end{pmatrix}$, $B = \begin{pmatrix} b_x \\ b_y \end{pmatrix}$

Verbindungsvektor $\vec{p} := B - A = \begin{pmatrix} b_x - a_x \\ b_y - a_y \end{pmatrix}$

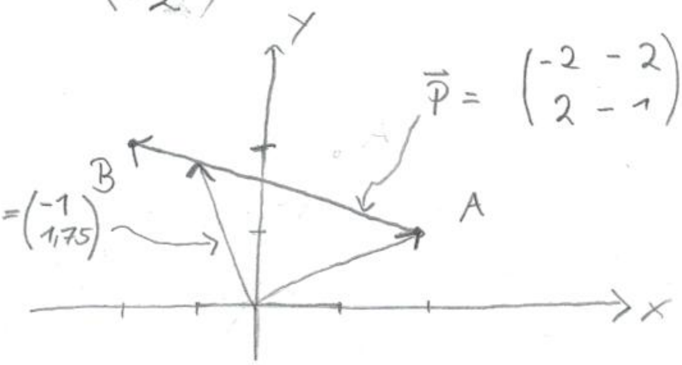
$$s(t) = A + t \cdot \vec{p}\,, \quad t \in [0,1]$$

$s(0) = A$, $s(1) = A + B - A = B$, „Strecke von A nach B"

Bsp: $A = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $B = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$

$\vec{p} = \begin{pmatrix} -2-2 \\ 2-1 \end{pmatrix} = \begin{pmatrix} -4 \\ 1 \end{pmatrix}$

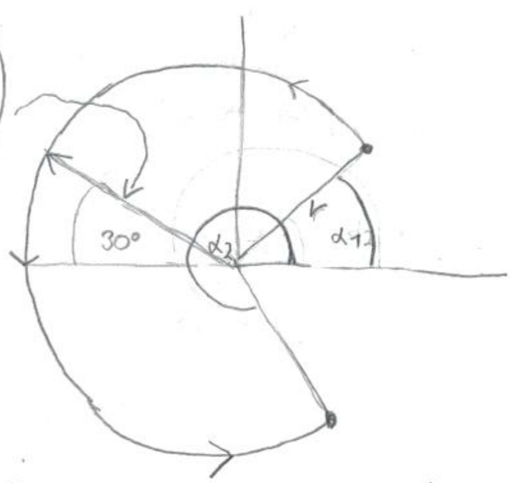
z.B. $s(0{,}75) = \begin{pmatrix} -1 \\ 1{,}75 \end{pmatrix}$
$\quad \; \shortparallel$
$A + 0{,}75\,\vec{p}$



$$S(t) = A + t\,\vec{p} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + t \begin{pmatrix} -4 \\ 1 \end{pmatrix} = \begin{pmatrix} 2-4t \\ 1+t \end{pmatrix}$$

(ii) Kreissegmente:

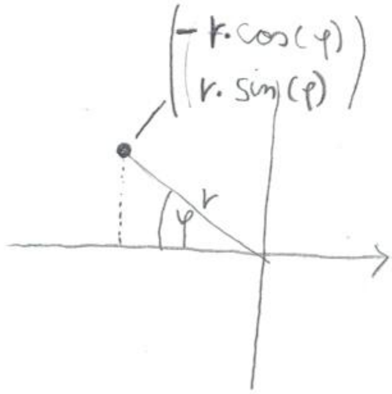
z.B. $s\left(5 \cdot \frac{\pi}{6}\right) = \begin{pmatrix} -\sqrt{3}/2 \cdot r \\ 1/2 \cdot r \end{pmatrix}$



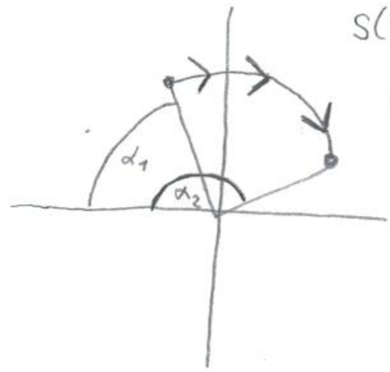
In Standardpolarkoordinaten: $s(t) = \begin{pmatrix} r \cdot \cos(t) \\ r \cdot \sin(t) \end{pmatrix}$, $t \in [\alpha_1, \alpha_2]$

andere Orientierung selbst herleiten :

z.B.



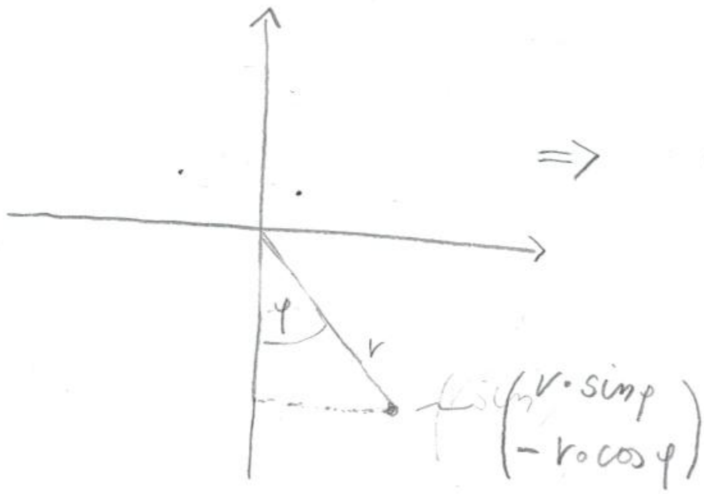
$\Rightarrow$



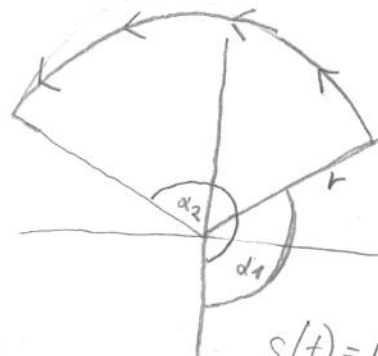
$s(t) = \begin{pmatrix} -r\cos t \\ r\sin t \end{pmatrix}$

$t \in [\alpha_1, \alpha_2]$

oder



$\Rightarrow$



$s(t) = \begin{pmatrix} r\cdot\sin t \\ -r\cdot\cos t \end{pmatrix}$

$t \in [\alpha_1, \alpha_2]$

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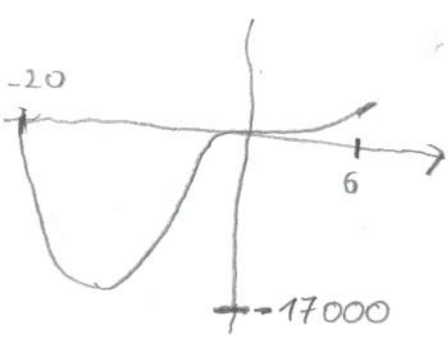
iii) <u>Funktionsgraphen</u> Ist $f : [a,b] \to \mathbb{R}$ eine Funktion, so parametrisiert $s(t) = \begin{pmatrix} t \\ f(t) \end{pmatrix}$, $t \in [a,b]$ den Graphen

z.B.

$f(x) = x^4 + 20x^3$, $x \in [-20, 6]$

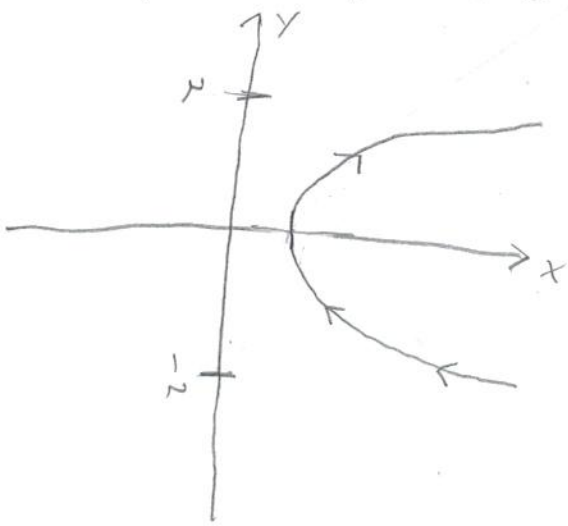


$s(t) = \begin{pmatrix} t \\ t^4 + 20t^3 \end{pmatrix}$ $t \in [-20, 6]$

oder Achsen vertauscht: $x = f(y)$, $y \in [a,b]$

$\Rightarrow S(t) = \begin{pmatrix} f(t) \\ t \end{pmatrix}$, $t \in [a,b]$

z.B. $x = y^2+1$, $x \in [-2,2]$



$\Rightarrow S(t) = \begin{pmatrix} t^2+1 \\ t \end{pmatrix}$