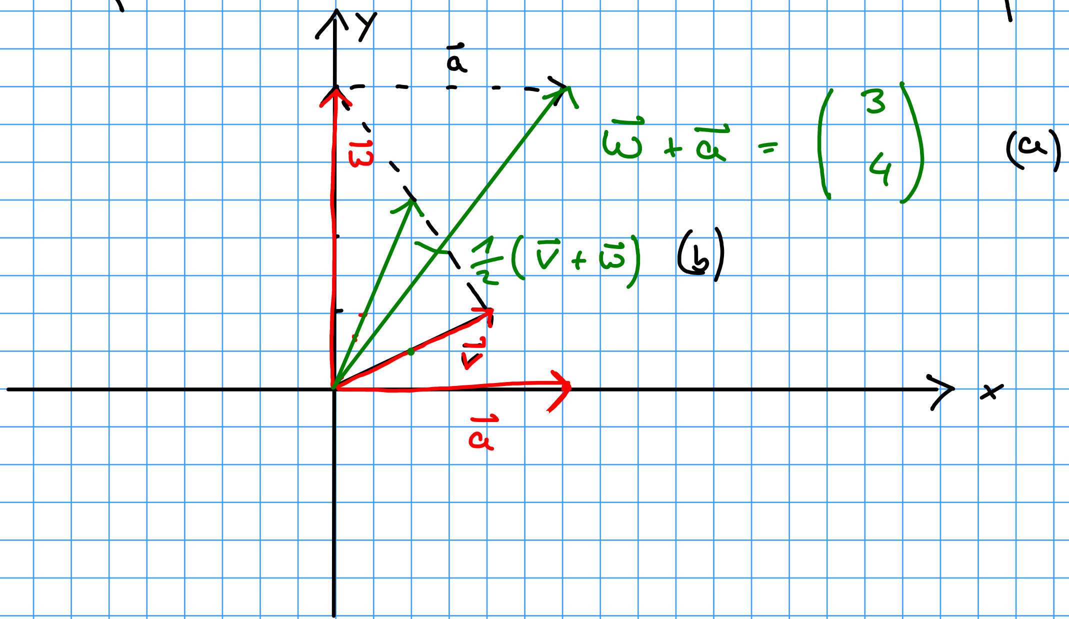
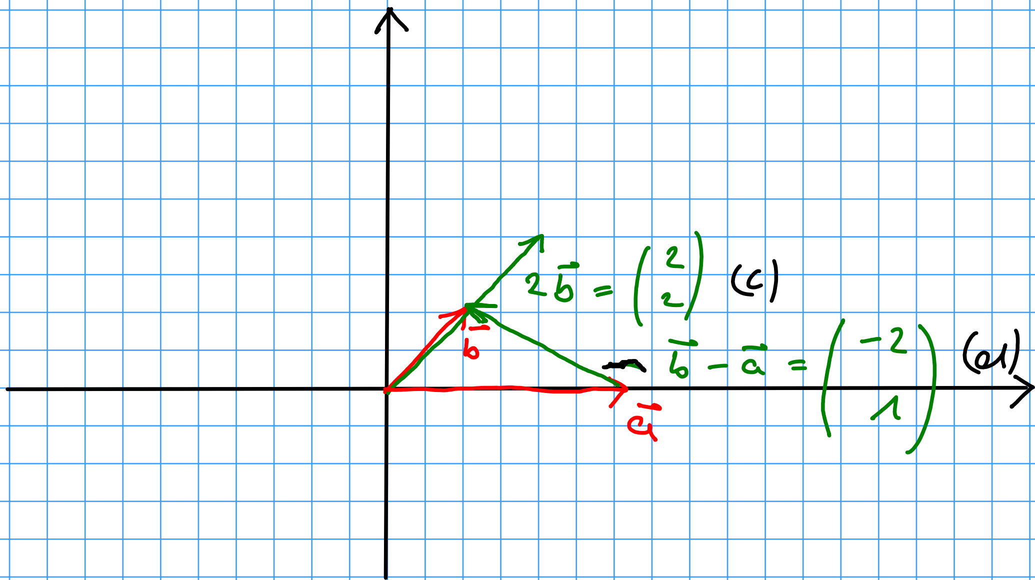
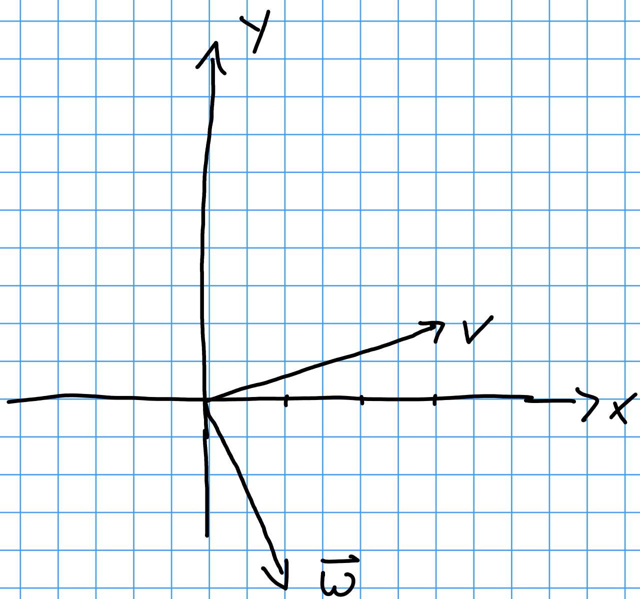
Aufgabe 1

$$\vec{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad \vec{w} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}, \quad \vec{a} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$





Aufgabe 2

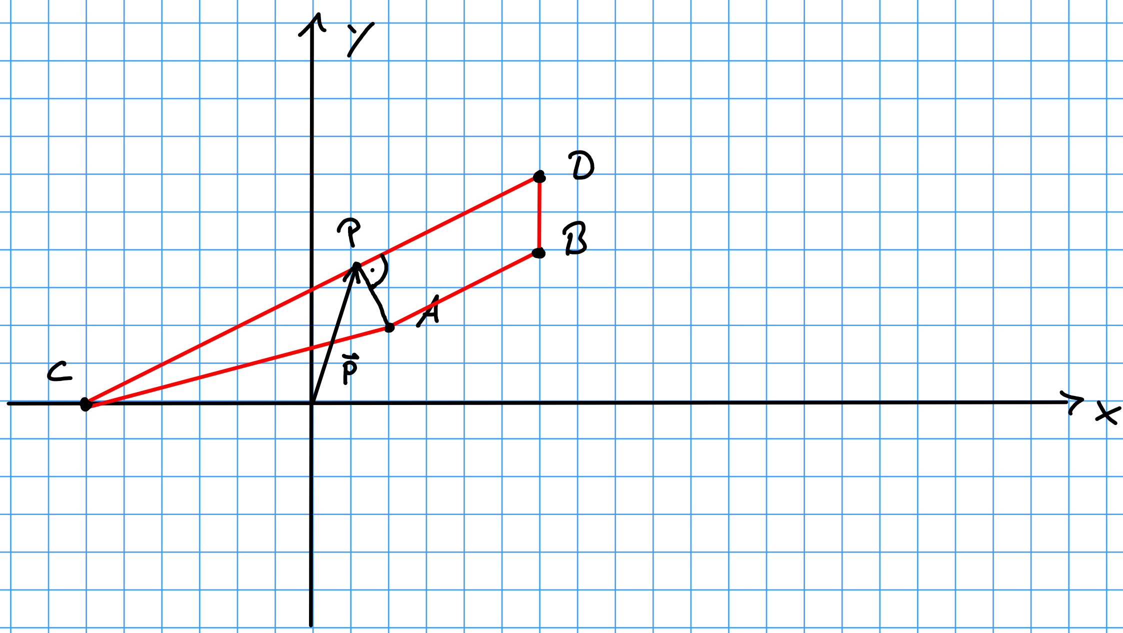


$\vec{v} \perp \vec{w} \iff \begin{pmatrix} 3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ \lambda \end{pmatrix} = 0$

$\iff 3 + \lambda = 0$

$\iff \underline{\underline{\lambda = -3}}$

Aufgabe 3



(a) $\vec{AB} = \vec{b} - \vec{a} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$\vec{CD} = \vec{d} - \vec{c} = \begin{pmatrix} 3 \\ 3 \end{pmatrix} - \begin{pmatrix} -3 \\ 0 \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$

$\vec{AB} = 3 \cdot \vec{CD}$ Also sind $\vec{AB}$ und $\vec{CD}$ parallel

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$\vec{CA} = \vec{a} - \vec{c} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{pmatrix} -3 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$

$\vec{DB} = \vec{b} - \vec{d} \quad \begin{pmatrix} 3 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$

$\vec{CA} \neq \lambda \cdot \vec{DB} \Rightarrow \vec{CA}$ und $\vec{DB}$ sind nicht parallel

(b) Weil $\overrightarrow{AB} \parallel \overrightarrow{CD}$ aber $\overrightarrow{CA} \nparallel \overrightarrow{DB}$

ist ABCD ein Trapez

(c) $$\vec{p} = \begin{pmatrix} -3 \\ 0 \end{pmatrix} + \lambda \cdot \begin{pmatrix} 6 \\ 3 \end{pmatrix} = \begin{pmatrix} -3+6\lambda \\ 3\lambda \end{pmatrix}$$

$$\Rightarrow \vec{p} - \vec{a} = \begin{pmatrix} -3+6\lambda \\ 3\lambda \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -4+6\lambda \\ 3\lambda-1 \end{pmatrix}$$

$$\vec{p} - \vec{a} \perp \overrightarrow{CD} \Leftrightarrow (\vec{p} - \vec{a}) \cdot \overrightarrow{CD} = 0$$

$$\Leftrightarrow \begin{pmatrix} -4+6\lambda \\ 3\lambda-1 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ 3 \end{pmatrix} = 0$$

$$\Leftrightarrow -24 + 36\lambda + 9\lambda - 3 = 0$$

$$\Leftrightarrow -27 + 45\lambda = 0$$

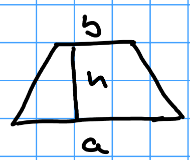
$$\Leftrightarrow \lambda = \cancel{\frac{27}{45}} = \frac{3}{5}$$

$$\Rightarrow \vec{p} - \vec{a} = \begin{pmatrix} -4 + 6 \cdot \frac{3}{5} \\ 3 \cdot \frac{3}{5} - 1 \end{pmatrix} = \begin{pmatrix} -\frac{20}{5} + \frac{18}{5} \\ \frac{9}{5} - \frac{5}{5} \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{2}{5} \\ \frac{4}{5} \end{pmatrix}$$

$$\Rightarrow |\vec{p} - \vec{a}| = \sqrt{\frac{4}{25} + \frac{16}{25}} = \underline{\underline{\frac{\sqrt{20}}{5}}} = \frac{2\sqrt{5}}{5}$$

(d) Flächeninhalt Trapez



$A = (a+b) \cdot h/2$

$$A = \left(|\overrightarrow{AB}| + |\overrightarrow{CD}|\right) \cdot |\vec{p} - \vec{a}| \cdot \frac{1}{2}$$

$$|\vec{AB}| = \left|\begin{pmatrix}2\\1\end{pmatrix}\right| = \sqrt{4+1} = \sqrt{5}$$

$$|\vec{CD}| = \left|\begin{pmatrix}6\\3\end{pmatrix}\right| = \sqrt{36+9} = \sqrt{45} = 3\sqrt{5}$$

$$\Rightarrow A = (\sqrt{5} + 3\cdot\sqrt{5}) \cdot \frac{2\sqrt{5}}{5} \cdot \frac{1}{2}$$

$$= 4 \cdot \cancel{\sqrt{5}} \cdot \frac{\cancel{2}\cancel{\sqrt{5}}}{\cancel{5}} \cdot \frac{1}{\cancel{2}} = \underline{\underline{4}}$$

Aufgabe 4

(a) „$E = g$"

$$\begin{pmatrix}3\\1\\1\end{pmatrix} + \lambda\begin{pmatrix}2\\1\\0\end{pmatrix} = \begin{pmatrix}2\\1\\0\end{pmatrix} + \mu\cdot\begin{pmatrix}0\\0\\1\end{pmatrix} + \tau\cdot\begin{pmatrix}1\\1\\0\end{pmatrix}$$

$$\begin{pmatrix}3+2\lambda\\1+\lambda\\1\end{pmatrix} = \begin{pmatrix}2+\tau\\1+\tau\\\mu\end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix}3\\1\\1\end{pmatrix} + \begin{pmatrix}2\lambda\\\lambda\\0\end{pmatrix} = \begin{pmatrix}2\\1\\0\end{pmatrix} + \begin{pmatrix}\tau\\\tau\\\mu\end{pmatrix} \quad \Big| -\begin{pmatrix}2\\1\\0\end{pmatrix} - \begin{pmatrix}2\lambda\\\lambda\\0\end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix}1\\0\\1\end{pmatrix} = \begin{pmatrix}\tau\\\tau\\\mu\end{pmatrix} - \begin{pmatrix}2\lambda\\\lambda\\0\end{pmatrix}$$

LGS $\quad 1\cdot\tau + 0\cdot\mu - 2\lambda = 1$

$1\cdot\tau + 0\cdot\mu - \lambda = 0$

$0\cdot\tau + 1\cdot\mu - 0\cdot\lambda = 1$

$$\rightsquigarrow \left(\begin{array}{ccc|c} 1 & 0 & -2 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \end{array}\right) \quad II - I$$

$$\rightsquigarrow \left(\begin{array}{ccc|c} 1 & 0 & -2 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 1 \end{array}\right)$$

$$\rightsquigarrow \left(\begin{array}{ccc|c} 1 & 0 & -2 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

$\Rightarrow \underline{\underline{\lambda = -1}}$

$\underline{\underline{\mu = 1}}$

$\Rightarrow \tau - 2\cdot(-1) = 1$

$\Rightarrow \tau + 2 = 1$

$\Rightarrow \underline{\underline{\tau = -1}}$

$\Rightarrow$ E und g schneiden sich

Schnittpunkt: (λ in g) $\quad \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} + (-1)\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$

$$= \underline{\underline{\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}}} \checkmark$$

(b) $\vec{q} = \begin{pmatrix} 0 \\ -1 \\ 3 \end{pmatrix}$

Abstand Q zu E

$$d = \frac{\left| \left(\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \right) \cdot (\vec{q} - \vec{p}) \right|}{\left| \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \right|}$$

<u>NR:</u> $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

$$\vec{q} - \vec{p} = \begin{pmatrix} 0 \\ -1 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \\ 3 \end{pmatrix}$$

Also $d = \dfrac{\left| \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ -2 \\ 3 \end{pmatrix} \right|}{\left| \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \right|} = \dfrac{0}{\sqrt{2}} = \underline{\underline{0}}$

$\Rightarrow$ Q liegt auf E

(c) $\vec{q} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \Rightarrow \vec{q} - \vec{p} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$

$$d = \frac{\left| \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \right|}{\left| \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \right|} = \underline{\underline{\frac{1}{\sqrt{2}}}}$$