Komplexe Zahlen

Aufgabe 1

(a) $z_1 \cdot z_2 + 5 = (1+3i)\cdot(2-i) + 5$

$= 2 - i + 6i + (3i)\cdot(-i) + 5$

$= 2 + 5i - 3 \cdot i^2 + 5 = 2 + 3 + 5 + 5i$

$= \underline{\underline{10 + 5i}}$

(b) $\frac{z_1^2}{\overline{z_2}} = \frac{(1+3i)^2}{2+i} = \frac{(1+3i)^2 \cdot (2-i)}{(2+i)(2-i)}$

$= \frac{(1 + 6i + 9 \cdot i^2)(2-i)}{4 + 2i - 2i - i^2} = \frac{(-8 + 6i)\cdot(2-i)}{4+1}$

$= \frac{-16 + 8i + 12i - 6i^2}{5} = \frac{-10 + 20i}{5}$

$= \underline{\underline{-2 + 4i}}$

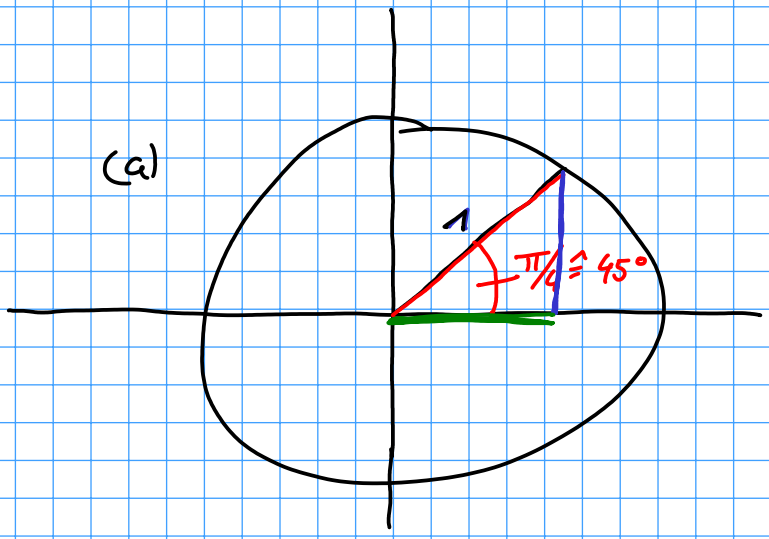
(c) $z_2 \cdot i^{10} = (2-i)\, i^{10} = (2-i)\cdot(i^2)^5$

$= (2-i)\cdot(-1)^5$

$= (2-i)\cdot(-1) = \underline{\underline{-2 + i}}$

Aufgabe 2

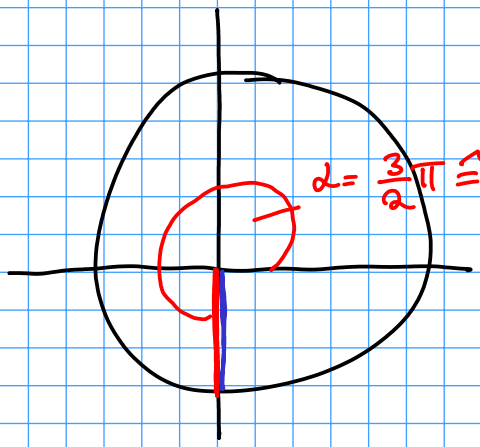
(a)



$\sin(\pi/4) = \frac{1}{\sqrt{2}}$

$\cos(\pi/4) = \frac{1}{\sqrt{2}}$

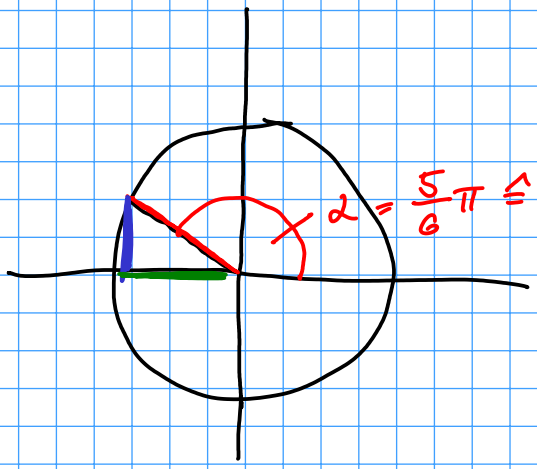
(b)



$\sin(\frac{3}{2}\pi) = -1$

$\cos(\frac{3}{2}\pi) = 0$

(c)



$\sin(\frac{5}{6}\pi) = \frac{1}{2}$

$\cos(\frac{5}{6}\pi) = -\frac{\sqrt{3}}{2}$

Aufgabe 3

$$4z^2 - 6z + 5 = 2z$$

$$\Leftrightarrow 4z^2 - 8z + 5$$

$$z_{1,2} = \frac{+8 \pm \sqrt{64 - 80}}{8} = \frac{8 \pm \sqrt{-16}}{8}$$

$$= \frac{8 \pm \sqrt{-1}\sqrt{16}}{8} = \frac{8 \pm i\,4}{8}$$

$$= 1 + \tfrac{1}{2}i,\ 1 - \tfrac{1}{2}i$$

$$\mathbb{L} = \left\{1 + \tfrac{1}{2}i,\ 1 - \tfrac{1}{2}i\right\}$$

Aufgabe 4

$$z^4 = \left(\sqrt{2} \cdot e^{\frac{3}{4}\pi \cdot i}\right)^8 = \left(\sqrt{2}\right)^8 \cdot \left(e^{3/4\,\pi \cdot i}\right)^8$$

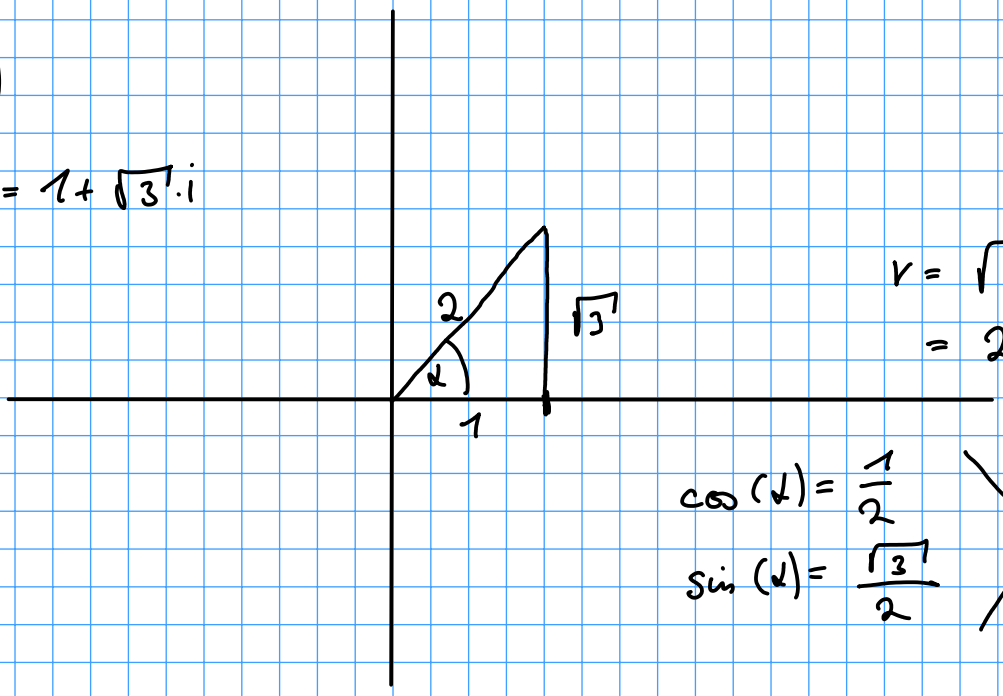
$$= \left(\left(\sqrt{2}\right)^2\right)^4 \cdot e^{3/4\,\pi \cdot i \cdot 8} = 2^4 \cdot e^{3 \cdot \pi \cdot i \cdot 2}$$

$$= 16 \cdot e^{6\pi i} = 16$$

$6\pi = 3 \cdot 2\pi \mathrel{\hat{=}} 3 \cdot 360°$

(b)

$\omega = 1 + \sqrt{3} \cdot i$



$r = \sqrt{1^2 + \sqrt{3}^2}$

$= 2$

$\cos(\alpha) = \frac{1}{2}$

$\sin(\alpha) = \frac{\sqrt{3}}{2}$

$\Rightarrow \alpha = \pi/3 \hat{=} 60°$

Also $\omega = 2 \cdot e^{\pi/3}$

$\Rightarrow \omega^6 = 2^6 \cdot e^{\pi/3 \cdot 6}$

$= 64 \cdot e^{2\pi} = 64 \cdot 1 = \underline{\underline{64}}$