Aufgabe 1

(a) $$\int_0^2 x^5 + \frac{1}{2}x^2\,dx = \int_0^2 x^5\,dx + \frac{1}{2}\int_0^2 x^2\,dx$$

$$= \left[\frac{1}{6}x^6\right]_0^2 + \frac{1}{2}\cdot\left[\frac{1}{3}x^3\right]_0^2$$

$$= \frac{1}{6}\left(2^6 - 0^6\right) + \frac{1}{2}\cdot\left(\frac{1}{3}\cdot 2^3 - 0^3\right)$$

$$= \frac{1}{\cancel{6}_3}\cdot 2^{\cancel{6}5} + \frac{1}{\cancel{2}}\cdot 2^{\cancel{3}2}\cdot\frac{1}{3}$$

$$= \frac{32}{3} + \frac{4}{3} = \frac{36}{3} = \underline{\underline{12}}$$

$$\int \frac{6}{x^2} - \frac{1}{x}\,dx = \int\frac{6}{x^2} - \int\frac{1}{x}\,dx$$

$$= 6\int x^{-2} - \ln(x)$$

$$= 6\cdot\frac{1}{-1}\cdot x^{-1} - \ln(x)$$

$$= \underline{\underline{-\frac{6}{x} - \ln(x)}}$$

Aufgabe 2

(a) $$\int_0^{\pi} (x+1)\cdot\sin(x)\,dx$$

$$\downarrow \qquad \uparrow$$

$$= \left[-(x+1)\cdot\cos(x)\right]_0^{\pi} - \int_0^{\pi} -\cos(x)\,dx$$

$$= -(\pi+1)\cdot\underbrace{\cos(\pi)}_{-1} - \left(-(0+1)\cdot\underbrace{\cos(0)}_{1}\right) + \int_0^{\pi}\cos(x)\,dx$$

$$= (\pi + 1) + 1 \cdot 1 + \left[\sin(x)\right]_0^{\pi}$$

$$= \pi + 2 + \left(\underbrace{\sin(\pi)}_{0} - \underbrace{\sin(0)}_{0} \right)$$

$$= \underline{\underline{\pi + 2}}$$

(b) $\int \ln(x)\,dx = \int \underset{\uparrow}{1} \cdot \underset{\downarrow}{\ln}(x)\,dx$

$$= x \cdot \ln(x) - \int x \cdot \frac{1}{x}\,dx$$

$$= x \cdot \ln(x) - \int 1\,dx$$

$$= \underline{\underline{x \cdot \ln(x) - x + C}}$$

<u>Aufgabe 3</u>

(a) $$\int_0^{1/3} \frac{12}{(3x+1)^5}\,dx \underset{u = 3x+1}{=} \int_{3\cdot 0+1}^{3\cdot 1/3+1} \frac{12}{u^5} \cdot \frac{1}{3}\,du$$

$$\frac{du}{dx} = 3 \Rightarrow dx = \frac{1}{3}du$$

$$= \int_1^2 \frac{4}{u^5}\,du = 4\int_1^2 u^{-5}\,du$$

$$= 4 \cdot \left[\frac{1}{-4} \cdot u^{-4}\right]_1^2 = -\frac{4}{4} \cdot \left[\frac{1}{u^4}\right]_1^2$$

$$= -1 \left(\frac{1}{2^4} - \frac{1}{1^4}\right)$$

$$= -1\left(\frac{1}{16} - 1\right) = -1\left(\frac{1}{16} - \frac{16}{16}\right)$$

$$= -1 \cdot \left(-\frac{15}{16}\right) = \underline{\underline{\frac{15}{16}}}$$

(b) $\int e^{3x^2+7} \; 2x \; dx$

$u = 3x^2 + 7$

$\frac{du}{dx} = 6x \Rightarrow dx = \frac{1}{6x} du$

$$= \int e^u \cdot \cancel{2x} \cdot \frac{1}{\underset{3}{\cancel{6x}}} du \Bigg|_{u = 3x^2+7}$$

$$= \int \frac{1}{3} e^u du \Bigg|_{u=3x^2+7} = \frac{1}{3} \int e^u du \Bigg|_{u=3x^2+7}$$

$$= \frac{1}{3} \cdot e^u \Bigg|_{u=3x^2+7} = \frac{1}{3} \cdot e^{3x^2+7}$$