Aufgabe 1

(a) $\int_0^2 x^5 + \frac{1}{2}x^2\,dx = \int_0^2 x^5\,dx + \frac{1}{2}\int_0^2 x^2\,dx$

$= \left[\frac{1}{6}x^6\right]_0^2 + \frac{1}{2}\cdot\left[\frac{1}{3}x^3\right]_0^2$

$= \frac{1}{6}\left(2^6 - 0^6\right) + \frac{1}{2}\cdot\left(\frac{1}{3}\cdot 2^3 - 0^3\right)$

$= \frac{1}{\cancel{6}_3}\cdot 2^{\cancel{6}5} + \frac{1}{\cancel{2}}\cdot 2^{\cancel{3}2}\cdot\frac{1}{3}$

$= \frac{32}{3} + \frac{4}{3} = \frac{36}{3} = \underline{\underline{12}}$

(b) $\int \frac{6}{x^2} - x\,dx = \int \frac{6}{x^2}\,dx - \int x\,dx$

$= 6\int x^{-2}\,dx - \frac{1}{2}x^2$

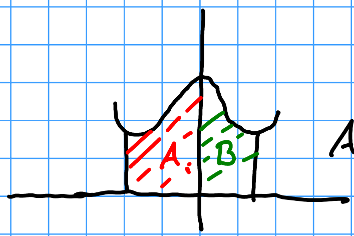
$= 6\cdot\frac{1}{-1}\cdot x^{-1} - \frac{1}{2}x^2$

$= \underline{\underline{-\frac{6}{x} - \frac{1}{2}x^2}}$

Aufgabe 2

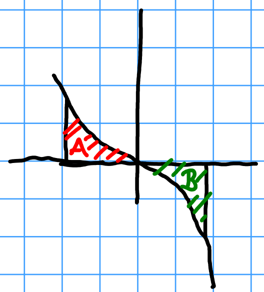
(i) wahr! Die Geschwindigkeit integriert ergibt die Strecke

(ii) wahr



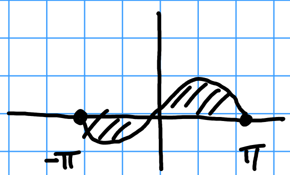
$A + B = 2B$

(iii) falsch



$A - B = 0$

(iv) falsch



$$\int_{-\pi}^{\pi} \sin(x)\,dx = 0$$

(v) wahr
$$\int_0^2 f'(x)\,dx = f(2) - f(0) = f(2)$$

(vi) wahr
$$\int_0^{10} g(x)\,dx = \int_0^{10} f(x) + 3 \quad dx$$
$$= \int_0^{10} f(x)\,dx + \int_0^{10} 3\,dx = \int f(x)\,dx + [3x]_0^{10}$$
$$= \int f(x)\,dx + 30$$

Aufgabe 3

$$\int_0^{\pi} (x+1) \cdot \sin(x)\,dx$$
$$\downarrow \qquad \uparrow$$

$$= \left[-(x+1)\cdot\cos(x)\right]_0^{\pi} - \int_0^{\pi} -\cos(x)\,dx$$

$$= -(\pi+1)\cdot\underbrace{\cos(\pi)}_{-1} - \left(-(0+1)\cdot\underbrace{\cos(0)}_{1}\right) + \int_0^{\pi} \cos(x)\,dx$$

$$= (\pi + 1) + 1 \cdot 1 + \left[\sin(x)\right]_0^{\pi}$$

$$= \pi + 2 + \left(\underbrace{\sin(\pi)}_{0} - \underbrace{\sin(0)}_{0}\right)$$

$$= \underline{\underline{\pi + 2}}$$

<u>Aufgabe 4</u>

Beschleunigung:

$$a(t) = 30 - 0{,}3 \cdot t$$

Geschwindigkeit:

$$v(t) = \int_0^t 30 - 0{,}3\, t' \, dt'$$

$$= \left[30 \cdot t'\right]_0^t - \left[0{,}3 \cdot \frac{1}{2} \cdot t'^2\right]_0^t = 30 \cdot t - \frac{3}{20} \cdot t^2$$

Strecke nach 100 Sekunden

$$S = \int_0^{100} 30 \cdot t - \frac{3}{20} t^2 \, dt$$

$$= 30 \int_0^{100} t \, dt - \frac{3}{20} \int_0^{100} t^2 \, dt$$

$$= 30 \cdot \left[\frac{1}{2} t^2\right]_0^{100} - \frac{3}{20} \cdot \left[\frac{1}{3} t^3\right]_0^{100}$$

$$= 15 \cdot 100^2 - \frac{1}{20} \cdot 100^3$$

$$= 150\,000 - \frac{1}{2} \cdot 100\,000 = 100\,000 \text{ m}$$

Aufgabe 5

(a) $$\int_0^{1/3} \frac{12}{(3x+1)^5}\,dx \underset{u=3x+1}{=} \int_{3\cdot 0+1}^{3\cdot 1/3+1} \frac{12}{u^5}\cdot\frac{1}{3}\,du$$

$$\frac{du}{dx}=3 \Rightarrow dx=\frac{1}{3}du$$

$$= \int_1^2 \frac{4}{u^5}\,du = 4\int_1^2 u^{-5}\,du$$

$$= 4\cdot\left[\frac{1}{-4}\cdot u^{-4}\right]_1^2 = -\frac{4}{4}\cdot\left[\frac{1}{u^4}\right]_1^2$$

$$= -1\left(\frac{1}{2^4}-\frac{1}{1^4}\right)$$

$$= -1\left(\frac{1}{16}-1\right) = -1\left(\frac{1}{16}-\frac{16}{16}\right)$$

$$= -1\cdot\left(-\frac{15}{16}\right) = \underline{\underline{\frac{15}{16}}}$$

(b) $$\int e^{3x^2+7}\,2x\,dx$$

$$u=3x^2+7$$

$$\frac{du}{dx}=6x \Rightarrow dx=\frac{1}{6x}du$$

$$= \int e^u\cdot\cancel{2x}\cdot\frac{1}{\cancel{6x}_{3}}\,du\ \Big|_{u=3x^2+7}$$

$$= \int \frac{1}{3}e^u\,du\Big|_{u=3x^2+7} = \frac{1}{3}\int e^u\,du\ \Big|_{u=3x^2+7}$$

$$= \frac{1}{3}\cdot e^u\Big|_{u=3x^2+7} = \underline{\underline{\frac{1}{3}\cdot e^{3x^2+7}}}$$