Aufgabe 1

(a) $f(x) = 3 \cdot x^6 + \frac{1}{2} x^2 + x + 8$

$f'(x) = (3x^6)' + \left(\frac{1}{2} x^2\right)' + (x)' + 8'$

$= 3 \cdot 6 \cdot x^5 + \frac{1}{2} \cdot 2 \cdot x^1 + 1 \cdot x^0 + 0$

$= \underline{\underline{18x^5 + x + 1}}$

(b) $f(x) = \sin(x) \cdot x + 2x$

$f'(x) = (\sin(x) \cdot x)' + (2x)'$

$= (\sin(x))' \cdot x + \sin(x) \cdot (x)' + 2$

$= \cos(x) \cdot x + \sin(x) \cdot 1 + 2$

$= \underline{\underline{x \cos(x) + \sin(x) + 2}}$

(c)

$f(x) = (x+1)^{12}$

$f'(x) = 12 \cdot (x+1)^{11} \cdot (x+1)'$

$= 12 \cdot (x+1)^{11} \cdot 1 = \underline{\underline{12 \cdot (x+1)^{11}}}$

(d) $f(x) = \cos(3x)$

$f'(x) = -\sin(3x) \cdot 3$

(e) $f(x) = e^{x^2+3x}$

$f'(x) = e^{x^2+3x} \cdot (x^2+3x)' = \underline{\underline{e^{x^2+3x} \cdot (2x+3)}}$

(f) $f(x) = \sqrt{x^2+1} = (x^2+1)^{1/2}$

$f'(x) = \frac{1}{2}(x^2+1)^{-1/2} \cdot 2x = \underline{\underline{\frac{x}{\sqrt{x^2+1}}}}$

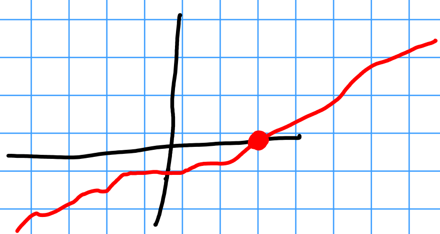
Aufgabe 2

(i) wahr. Die Ableitung der Strecke ist die Geschwindigkeit (Änderungsrate)

(ii) falsch $f(x) = 3x$ hat keinen Extremwert

(iii) falsch $f(x) = -x^2+2$ hat ein lok Max aber kein lok. Min

(IV) wahr



(V) wahr (Mittelwertsatz)

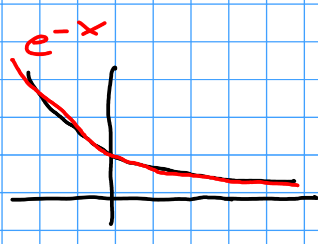
irgendwo immer waagerechte Tangente

wahr

(VI) $f(x) = \frac{1}{x}$, $f'(x) = -\frac{1}{x^2}$, $f''(x) = \frac{2}{x^3}$

$f''(2) = \frac{2}{8} = \frac{1}{4}$ Taylorpolynom hat selbe zweite Ableitung am Entwicklungspunkt wie Funktion

(VII) falsch $f(x)\ e^{-x}$ hat überall negative Ableitung und $f(0) = e^0 = 1$ aber f hat keine Nst:



(VIII) falsch

$$f'(t) = -p(t) \cdot \frac{1}{10}$$

Aufgabe 3

(i) $f(x) = x^2 - 3x + 7$ $\qquad$ $f(2) = 4 - 6 + 7 = 5$

$f'(x) = 2x - 3$ $\qquad$ $f'(2) = 1$

$$\Rightarrow \quad T_{2,1}(x) = f(2) + f'(2) \cdot (x-2)$$
$$= 5 + 1 \cdot (x-2) = 5 + x - 2 = \underline{\underline{x+3}}$$

(ii) $f(x) = \frac{1}{x}$ $\qquad$ $f(1) = 1$

$f'(x) = -\frac{1}{x^2}$ $\qquad$ $f'(1) = -1$

$f''(x) = \frac{2}{x^3}$ $\qquad$ $f''(1) = 2$

$$\Rightarrow T_{1,2}(x) = f(1) + f'(1) \cdot (x-1) + \frac{f''(1)}{2} \cdot (x-1)^2$$
$$= 1 - 1 \cdot (x-1) + \frac{2}{2} \cdot (x-1)^2$$
$$= 1 - x + 1 + x^2 - 2x + 1$$
$$= x^2 - 3x + 3$$

Aufgabe 4

$$a + 2b = 60 \quad \Rightarrow \quad b = \frac{60 - a}{2} = 30 - \frac{a}{2}$$

maximiere $a \cdot b = a\left(30 - \frac{a}{2}\right)$

$$= 30a - \frac{a^2}{2}$$

Flächeninhalt $= f(a) = 30a - \frac{a^2}{2}$

Falls $f(a)$ lok. Max $\Rightarrow f'(a) = 0$

$$f'(a) = 30 - a \overset{!}{=} 0 \iff a = 30$$

ist $a = 30$ lok Max ?

$$f''(a) = -1 \quad \Rightarrow \quad \text{lok. Max}$$

Also $a = 30$ und $b = 15$

Aufgabe 5

(a) $f(x) = x^4 - 2x^2$

$f'(x) = 4x^3 - 4x = 4x(x^2-1) = 4x(x-1)(x+1)$

$f''(x) = 12x^2 - 4$

Extremwerte $f'(x) = 0 \iff 4x(x-1)(x+1) = 0$

$\iff x = 0, 1, -1$

$f''(0) = -4 \Rightarrow$ lok. Max

$f''(1) = f''(-1) = 8 \Rightarrow$ lok Min

$(0, 0)$ lok Max

$(-1, -1)$ lok Min

$(1, -1)$ lok Min

(b) $f(x) = 2x^3 - 3x^2 + 4$

$f'(x) = 6x^2 - 6x = 6 \cdot x(x-1)$ $f''(x) = 12x - 6$

Extremwerte $f'(x) = 0 \iff 6x(x-1) = 0$

$\iff x = 0, 1$

$f''(0) = -6 \Rightarrow$ lok. Max $(0, 4)$

$f''(1) = 6 \Rightarrow$ lok Min $(1, 3)$

(c) $f(x) = e^{-x^2-2x+1}$

$$f'(x) = e^{-x^2-2x+1} \cdot (-2x-2)$$

$$f''(x) = e^{-x^2-2x+1} \cdot (-2x-2)^2 + e^{-x^2-2x+1} (-2)$$

$$= e^{-x^2-2x+1} \left(4x^2+8x+4-2\right)$$

$$= e^{-x^2-2x+1} \left(4x^2+8x+2\right)$$

Extremwerte

$$f'(x) = 0 \iff e^{-x^2-2x+1} \cdot (-2x-2) = 0$$

$$\iff -2x-2 = 0$$

$$\iff -2 = 2x \iff x = -1$$

$$f''(-1) = \underbrace{e^{\cdots}}_{>0} \cdot (4-8+2) = e^{\cdots} \cdot (-2) < 0$$

$\Rightarrow$ lok. Max $(-1, e^2)$

Aufgabe 6

(a) $f(x) = x^3 - 6x^2 + 8x$

$f'(x) = 3x^2 - 12x + 8$

$f''(x) = 6x - 12$

$f'(x) = 0 \iff 3x^2 - 12x + 8 = 0 \iff x_{1,2} = \frac{12 \pm \sqrt{144 - 96}}{6}$

$\iff 2 \pm \frac{4\sqrt{3}}{6} = 2 \pm \frac{2}{\sqrt{3}}$

$f''(2 + \frac{2}{\sqrt{3}}) > 0 \Rightarrow$ lok. Min bei $\left(2 + \frac{2}{\sqrt{3}}, -\frac{16 \cdot \sqrt{3}}{9}\right) \approx (3{,}15 / -3{,}08)$

$f''(2 - \frac{2}{\sqrt{3}}) < 0 \Rightarrow$ lok. Max bei $\left(2 - \frac{2}{\sqrt{3}}, \frac{16\sqrt{3}}{9}\right) \approx (0{,}85 / +3{,}08)$

- Keine Symmetrie
- Nullstellen :

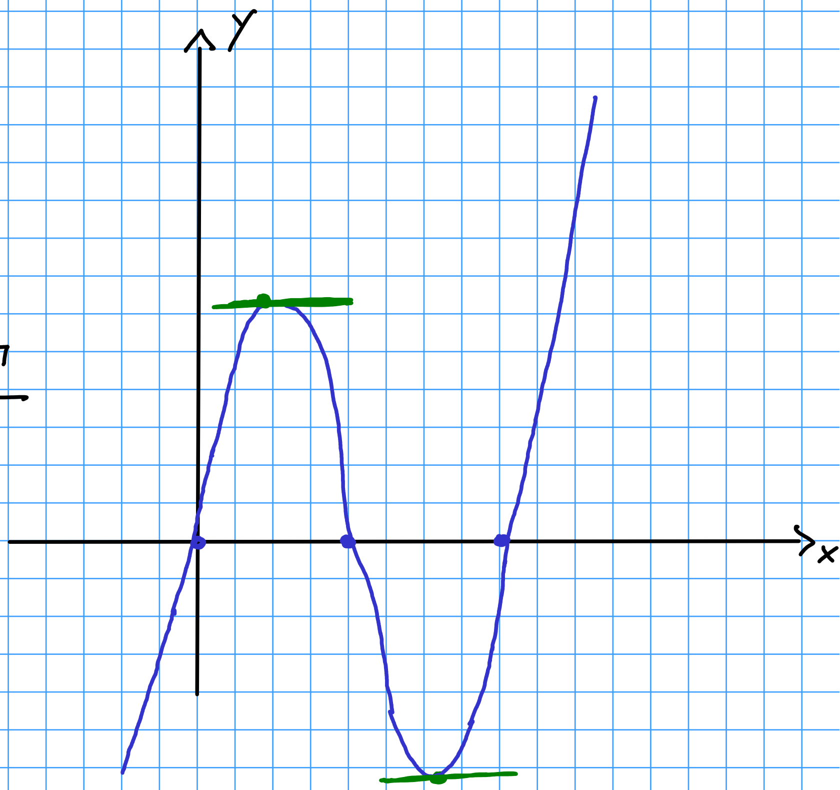
$x^3 - 6x^2 + 8x = 0$

$\iff x(x^2 - 6x + 8) = 0$

$\iff x_1 = 0 \quad x_{2,3} = \frac{6 \pm \sqrt{36 - 32}}{2}$

$= 3 \pm 1$

$= 4, 2$



- Verhalten für $x \to \pm\infty$

$\lim_{x \to \infty} x^3 - 6x^2 + 8x = \infty$

$\lim_{x \to -\infty} x^3 - 6x^2 + 8x = -\infty$

(lim $\hat{=}$ Limes der Grenzwert)

(b) $\frac{x^2}{x+1} = f(x)$ $\quad f'(x) = \frac{2x(x+1) - x^2}{(x+1)^2} = \frac{2x^2+2x-x^2}{(x+1)^2}$

$$= \frac{x^2+2x}{(x+1)^2}$$

$$f''(x) = \frac{(2x+2)(x+1)^2 - (x^2+2x)\cdot 2\cdot(x+1)}{(x+1)^4}$$

$$= \frac{(2x+2)(x^2+2x+1) - (x^2+2x)(2x+2)}{(x+1)^4}$$

$$= \frac{2x^3+4x^2+2x+2x^2+4x+2-(2x^3+2x^2+4x^2+4x)}{(x+1)^4} = \frac{2x+2}{(x+1)^4}$$

Nst. $f(x)=0 \iff x=0$

Schnitt mit y-Achse $f(0)=0$

$\lim\limits_{x\to\infty} f(x) = \infty$

$\lim\limits_{x\to-\infty} f(x) = -\infty$

Pol $x=-1$

$\lim\limits_{x\to -1^-} f(x) = -\infty$

$\lim\limits_{x\to -1^+} f(x) = \infty$

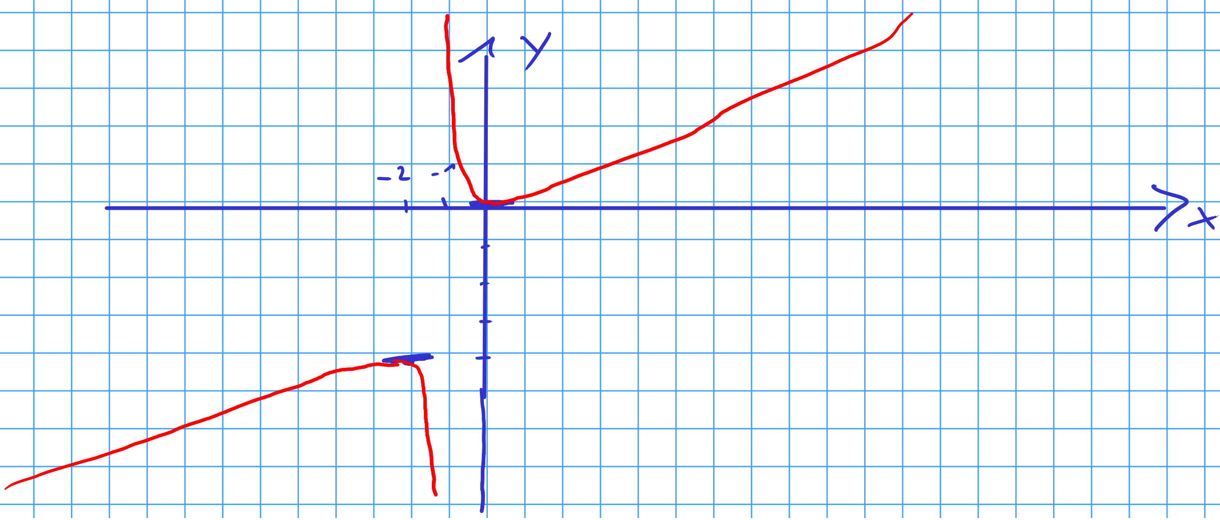
Extremwerte

$f'(x)=0 \iff x^2+2x = 0 \iff x(x+2)=0$

$\iff x = 0, -2$

$f''(0) = 2 > 0 \Rightarrow$ lok Min bei $(0,0)$

$f''(-2) = -2 < 0 \Rightarrow$ lok Max bei $(-2,-4)$



(c) $f(x) = (x+1)\cdot e^{-x}$

$f'(x) = e^{-x} - (x+1)\,e^{-x} = e^{-x}\left(1-(x+1)\right)$

$= -xe^{-x}$

$f''(x) = -e^{-x} + xe^{-x} = e^{-x}(x-1)$

Extremwerte $\quad f'(x) = 0 \Leftrightarrow x = 0$

$f''(0) = -1 \Rightarrow$ lok Max bei $(0,1)$

Verhalten für $x \to \pm\infty$

$$\lim_{x\to\infty} (x+1)\cdot e^{-x} = \lim_{x\to\infty} \frac{x+1}{e^{x}} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{x\to\infty} \frac{1}{e^{x}} = 0$$

$$\lim_{x\to-\infty} (x+1)\,e^{-x} = -\infty$$

Nullstelle bei $x = -1$