Aufgaben Potenzen und Summen

Aufgabe 1

(a) $$\sum_{k=0}^{3} \frac{5}{k^2+1} = \frac{5}{0^2+1} + \frac{5}{1^2+1} + \frac{5}{2^2+1} + \frac{5}{3^2+1}$$

$$= \frac{5}{1+1} + \frac{5}{1+1} + \frac{5}{4+1} + \frac{5}{9+1}$$

$$= \frac{5}{2} + \frac{5}{2} + \frac{5}{5} + \frac{5}{10}$$

(b) $$\sum_{k=1}^{4} \frac{2}{2k-1} \cdot x^{k-1} = \frac{2}{2\cdot 1-1} \cdot x^{1-1} + \frac{2}{2\cdot 2-1} \cdot x^{2-1}$$

$$+ \frac{2}{2\cdot 3-1} \cdot x^{3-1} + \frac{2}{2\cdot 4-1} \cdot x^{4-1}$$

$$= \frac{2}{1} \cdot \underbrace{x^0}_{=1} + \frac{2}{3} \cdot x^1 + \frac{2}{5} \cdot x^2 + \frac{2}{7} \cdot x^3$$

$$= 2 + \frac{2}{3}x + \frac{2}{5}x^2 + \frac{2}{7}x^3$$

Aufgabe 2

(a) $$\frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \frac{1}{16} = \frac{3}{2} + \frac{3}{2\cdot 2} + \frac{3}{2\cdot 2\cdot 2} + \frac{3}{2\cdot 2\cdot 2\cdot 2}$$

$$= \frac{3}{2^1} + \frac{3}{2^2} + \frac{3}{2^3} + \frac{3}{2^4}$$

$$= \sum_{k=1}^{4} \frac{3}{2^k}$$

(b) $2x + 4x^2 + 6x^3 + 8x^4 = 2\cdot 1\cdot x^1 + 2\cdot 2\cdot x^2 + 3\cdot 2\cdot x^3 + 4 2\cdot x^4$

$= \sum_{k=1}^{4} 2\cdot k\cdot x^k$

Aufgabe3

(a) $1^3 + 2^3 = 1 + 8 = 9$

(b) $(1+2)^3 = 3^3 = 27$

(c) $2^{(3^2)} = 2^9 = 2^4\cdot 2^4\cdot 2 = 16\cdot 16\cdot 2$

$= 512$

(d) $\left(2^3\right)^2 = 8^2 = 64$

(e) $(3^0 + 2^0)^2 = (1+1)^2 = 2^2 = 4$

(f) $\left(\frac{1}{2}\right)^{-3} = \frac{1}{\left(\frac{1}{2}\right)^3} = \frac{1}{\frac{1^3}{2^3}} = \frac{1}{\frac{1}{8}} = 8$

(g) $\left(-\frac{1}{2}\right)^3 = (-1)^3\cdot\left(\frac{1}{2}\right)^3 = -1\cdot\frac{1^3}{2^3} = -\frac{1}{8}$

Aufgabe 4

(a) $\left(-2^2\right)^3 = (-4)^3 = -64$

(b) $\left(2^{-2}\right)^3 = 2^{-2\cdot 3} = 2^{-6} = \frac{1}{2^6} = \frac{1}{64}$

(c) $10^3\cdot 10^{-5} = 10^{3-5} = 10^{-2} = \frac{1}{100}$

(d) $x^{m-2}\cdot x^{m+2} = x^{m-2+m+2} = x^{2m}$

(e) $\left(3^n + 3^{-n}\right)^2 \underset{\uparrow}{=} \left(3^n\right)^2 + 2\cdot 3^n\cdot 3^{-n} + \left(3^{-n}\right)^2$

binom. Formel

$= 3^{2\cdot n} + 2\cdot 3^{n-n} + 3^{-n\cdot 2} = 3^{2n} + 2 + 3^{-2n}$

(f) $(x+y)^2 \cdot (x^3 - y)$

$= (x^2 + 2xy + y^2)(x^3 - y) = x^2 \cdot x^3 + 2xy\cdot x^3 + y^2 \cdot x^3$
$\quad - x^2 \cdot y - 2xy\cdot y - y^2 \cdot y$

$= x^{2+3} + 2x^{1+3} \cdot y + y^2 x^3 - x^2 y - 2xy^{1+1} - y^{2+1}$

$= \underline{\underline{x^5 + 2x^4 y + y^2 x^3 - x^2 y - 2xy^2 - y^3}}$

Aufgabe 5

(a) $\underbrace{5000\,000}_{\text{6 Nullen}} = 5 \cdot 10^6$

(b) $\underbrace{0{,}0000123}_{\text{5 Verschiebungen}} = 1{,}23 \cdot 10^{-5}$

(c) $2{,}2 \cdot 10^{-12} - 1{,}2 \cdot 10^{-13}$

$= 2{,}2 \cdot 10^{-12} - 1{,}2 \cdot 10^{-12} \cdot 10^{-1}$

$= 2{,}2 \cdot 10^{-12} - 0{,}12 \cdot 10^{-12}$

$= (2{,}2 - 0{,}12) \cdot 10^{-12} = 2{,}08 \cdot 10^{-12}$

Aufgabe 6

(a) $c = 3 \cdot 10^{10}\ \text{cm}/\text{s}$

$\boxed{1\text{m} = 10^2 \text{cm} \Rightarrow 10^{-2}\text{m} = 1\text{cm}}$

$\Rightarrow$

$$3 \cdot 10^{10}\, cm/s = 3 \cdot 10^{10} \cdot \frac{10^{-2}\, m}{s}$$

$$= 3 \cdot 10^{10-2}\, m/s = \underline{\underline{3 \cdot 10^{8}\, m/s}}$$

$$\boxed{1h = 60 \cdot 60\, s = 3600\, s}$$

$$3 \cdot 10^{8}\, \frac{m}{s} = 3 \cdot 10^{8} \cdot \frac{m}{\frac{1}{3600} h}$$

$$= 3 \cdot 10^{8} \cdot 3600\, m/h$$

$$= 3 \cdot 36 \cdot 10^{8} \cdot 10^{2} = \underline{\underline{108 \cdot 10^{10}\, m/h}}$$

$$3 \cdot 10^{8}\, \frac{m}{s} = 3 \cdot 10^{8}\, \frac{10^{-3}\, km}{\frac{1}{60}\, min} = 3 \cdot 10^{8} \cdot 10^{-3} \cdot 60\, km/min$$

$$= \underline{\underline{180 \cdot 10^{5}\, km/min}}$$

(b)

Strecke

$$1a = 60 \cdot 60 \cdot 24 \cdot 365\, s$$

$$s = 3 \cdot 10^{8}\, \frac{m}{s} \cdot 60 \cdot 60 \cdot 24 \cdot 365 = \underline{\underline{9,4608 \cdot 10^{15}}}$$

$$\left(\begin{array}{l} = 94608 \cdot 10^{-4} \cdot 10^{15} \\ = 94608 \cdot 10^{11} \end{array} \right)$$